

New Results on GDH sum and Spin Structure at Low Q^2



Vincent Sulkosky

Massachusetts Institute of Technology

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Introduction

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 - Precise measurement of moments of spin structure functions at low Q^2 , 0.02 to 0.25 GeV² for the neutron and ${}^3\text{He}$.

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 - Cover an unmeasured region of kinematics to check theoretical calculations (Chiral Perturbation Theory).
 - Data from experiment E94-010 covered the transition region (0.1 to 0.9 GeV 2) from non-perturbative to perturbative QCD.
 - Preliminary results are available and final results are expected soon.

Inclusive Electron Scattering

Energy transfer:

$$\nu = E - E'$$

Momentum transfer:

$$\vec{q} = \vec{k} - \vec{k}'$$

4-momentum transfer squared:

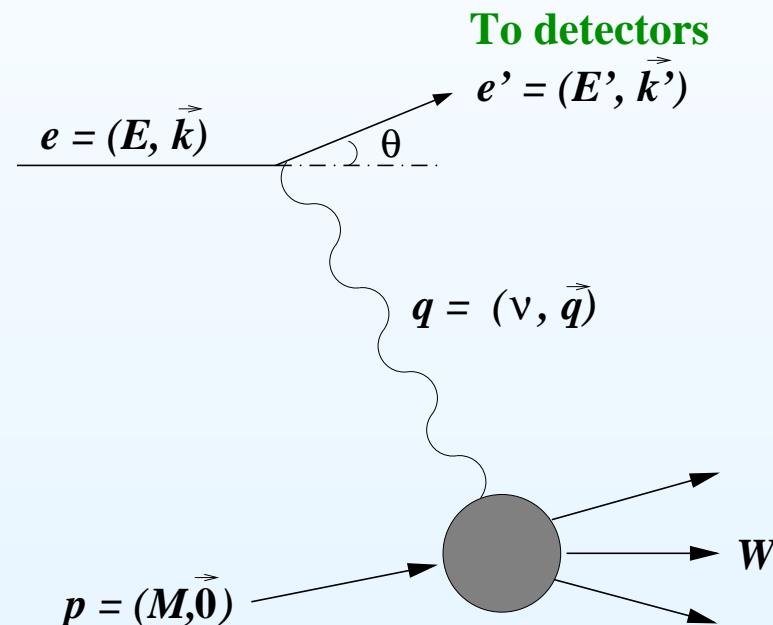
$$Q^2 = -q^2 = 4EE' \sin^2 \frac{\theta}{2}$$

Invariant Mass:

$$W^2 = M^2 + 2M\nu - Q^2$$

Bjorken variable:

$$x = \frac{Q^2}{2M\nu}$$



Inclusive Cross Sections

- structure functions:

g_1 and g_2 (quark polarizations)
or σ_{TT} and σ_{LT}

- Polarized cross sections:

$$\Delta\sigma_{||} = \frac{d^2\sigma_{\downarrow\uparrow}}{dE'd\Omega} - \frac{d^2\sigma_{\uparrow\uparrow}}{dE'd\Omega} = K \left[(E + E' \cos \theta) g_1(x, Q^2) - \left(\frac{Q^2}{\nu} \right) g_2(x, Q^2) \right]$$

$$\begin{aligned} \Delta\sigma_{\perp} &= \frac{d^2\sigma_{\downarrow\Rightarrow}}{dE'd\Omega} - \frac{d^2\sigma_{\uparrow\Rightarrow}}{dE'd\Omega} = KE' \sin \theta [g_1(x, Q^2) + \frac{2E}{\nu} g_2(x, Q^2)] \\ K &= \frac{4\alpha^2}{M\nu Q^2} \frac{E'}{E} \end{aligned}$$

$\downarrow\uparrow$ are for electron spin, $\uparrow\Rightarrow$ are for target spin direction

Inclusive Cross Sections

- structure functions:

g_1 and g_2 (quark polarizations)

or σ_{TT} and σ_{LT}

- Virtual photon-nucleon polarized cross sections:

$$2\sigma_{TT}(x, Q^2) = \sigma_{1/2}(x, Q^2) - \sigma_{3/2}(x, Q^2)$$

$$= \frac{8\pi^2\alpha}{M\textcolor{brown}{K}} \left[g_1(x, Q^2) - \frac{4M^2x^2}{Q^2} g_2(x, Q^2) \right]$$

$$\sigma_{LT}(x, Q^2) = \frac{4\pi^2\alpha}{M\textcolor{brown}{K}} [g_1(x, Q^2) + g_2(x, Q^2)]$$

$\textcolor{brown}{K}$: virtual photon flux

$\sigma_{1/2}, \sigma_{3/2}$: electroproduction cross sections

Gerasimov-Drell-Hearn (GDH) Sum Rule ($Q^2 = 0$)

$$I_{\text{GDH}} = \int_{\nu_{\text{th}}}^{\infty} \frac{\sigma_{\frac{1}{2}}(\nu) - \sigma_{\frac{3}{2}}(\nu)}{\nu} d\nu = -2\pi^2 \alpha \left(\frac{\kappa}{M} \right)^2$$

- Circularly **polarized photons** incident on a longitudinally polarized spin- $\frac{1}{2}$ target.
- $\sigma_{\frac{1}{2}}$ ($\sigma_{\frac{3}{2}}$) **photoabsorption cross section** with photon helicity parallel (anti-parallel) to the target spin.
- The sum rule is related to the target's **anomalous part of the magnetic moment** κ .
- Solid theoretical predictions based on general principles: Lorentz invariance, gauge invariance, unitarity and causality.

GDH Measurements

The sum rule is **valid for any target** with a given spin ($S = \frac{1}{2}, 1, \frac{3}{2}, \dots$).

	$M[\text{GeV}]$	Spin	κ	$I_{\text{GDH}}[\mu \text{ b}]$
Proton	0.938	$\frac{1}{2}$	1.79	-204.8
Neutron	0.940	$\frac{1}{2}$	-1.91	-233.2
Deuteron	1.876	1	-0.14	-0.65
Helium-3	2.809	$\frac{1}{2}$	-8.38	-498.0

- Proton sum rule was verified (**7%**): Mainz, Bonn and LEGS.
- Measurements for the **neutron** (deuteron) are in progress.

Generalized GDH Integral ($Q^2 > 0$)

$$I(Q^2) = \int_{\nu_{\text{th}}}^{\infty} \left[\sigma_{\frac{1}{2}}(\nu, Q^2) - \sigma_{\frac{3}{2}}(\nu, Q^2) \right] \frac{d\nu}{\nu}$$

$$\sigma_{1/2} - \sigma_{3/2} = \frac{8\pi^2\alpha}{MK} \left[g_1(\nu, Q^2) - \left(\frac{Q^2}{\nu^2} \right) g_2(\nu, Q^2) \right]$$

- Photoproduction cross sections \Rightarrow electroproduction cross sections.
- X.-D. Ji and J. Osborne, J. Phys. **G27**, 127 (2001)
Forward virtual compton scattering amplitudes: $S_1(Q^2)$, $S_2(Q^2)$.

$$S_1(Q^2) = \frac{8}{Q^2} \int_0^1 g_1(x, Q^2) dx = \frac{8}{Q^2} \Gamma_1(Q^2)$$

- Based on same general principles: Lorentz invariance, gauge invariance, unitarity and causality.

First Moments of g_1 and g_2

$$\Gamma_1 = \int_0^1 g_1(x, Q^2) dx$$

$$\Gamma_2 = \int_0^1 g_2(x, Q^2) dx$$

Bjorken Sum Rule ($Q^2 \rightarrow \infty$)

$$\Gamma_1^p - \Gamma_1^n = \frac{g_A}{6}$$

J.D. Bjorken, Phys. Rev. 148, 1467 (1966)

- $g_A = (1.2695 \pm 0.0029)$: the **nucleon axial charge**.
- The sum rule has been measured and agrees with expected value (at the 10% level).

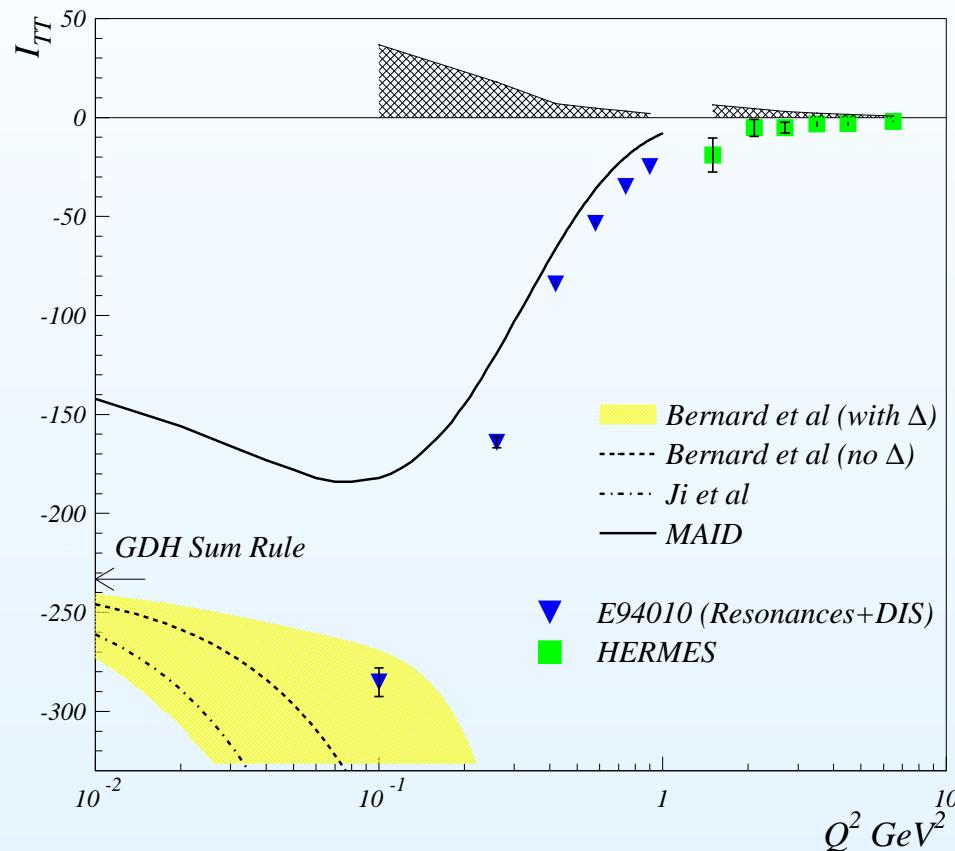
Importance of the Generalized GDH Sum Rule



- Constrained at the two ends of the Q^2 spectrum by known sum rules: GDH ($Q^2 = 0$) and Bjorken ($Q^2 \rightarrow \infty$).
- Generalized GDH Integral is **calculable at any Q^2** .
- Compare theoretical calculations to experimental measurements over the measurable Q^2 range.
- Tool to **study transition from non-perturbative to perturbative QCD**.

Neutron and ^3He GDH Results

Neutron

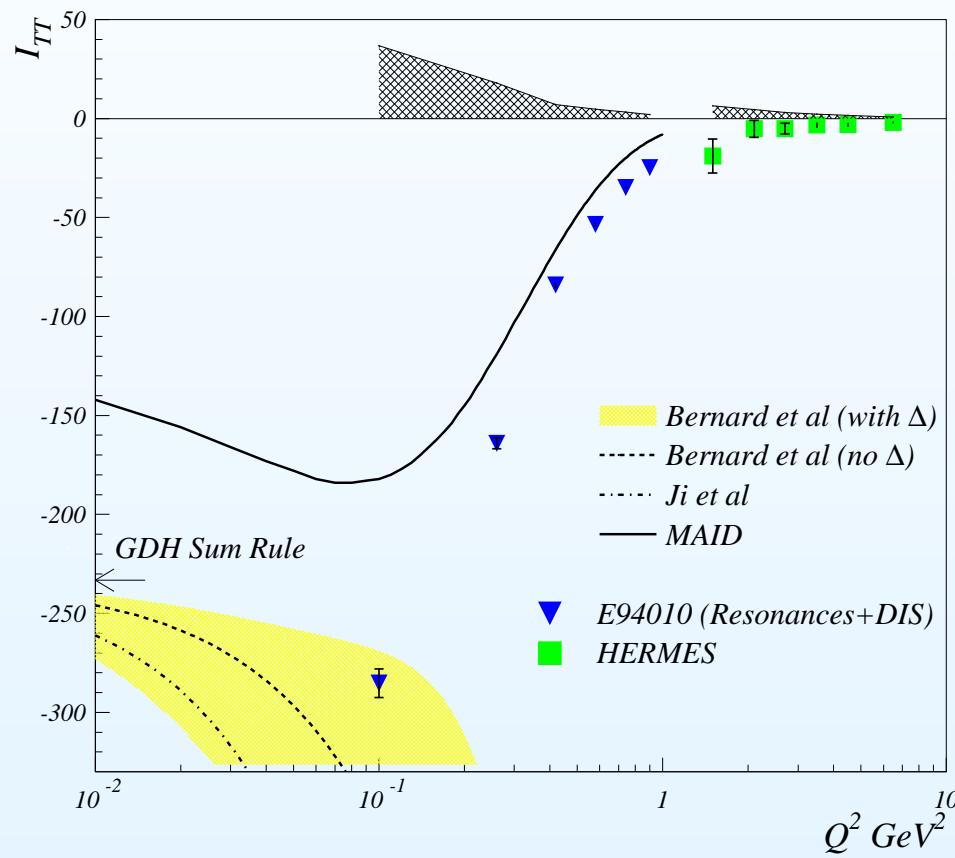


M. Amarian *et al.*, PRL 89, 242301 (2002)

MAID: **phenomenological model** with only resonance contributions.

Neutron and ^3He GDH Results

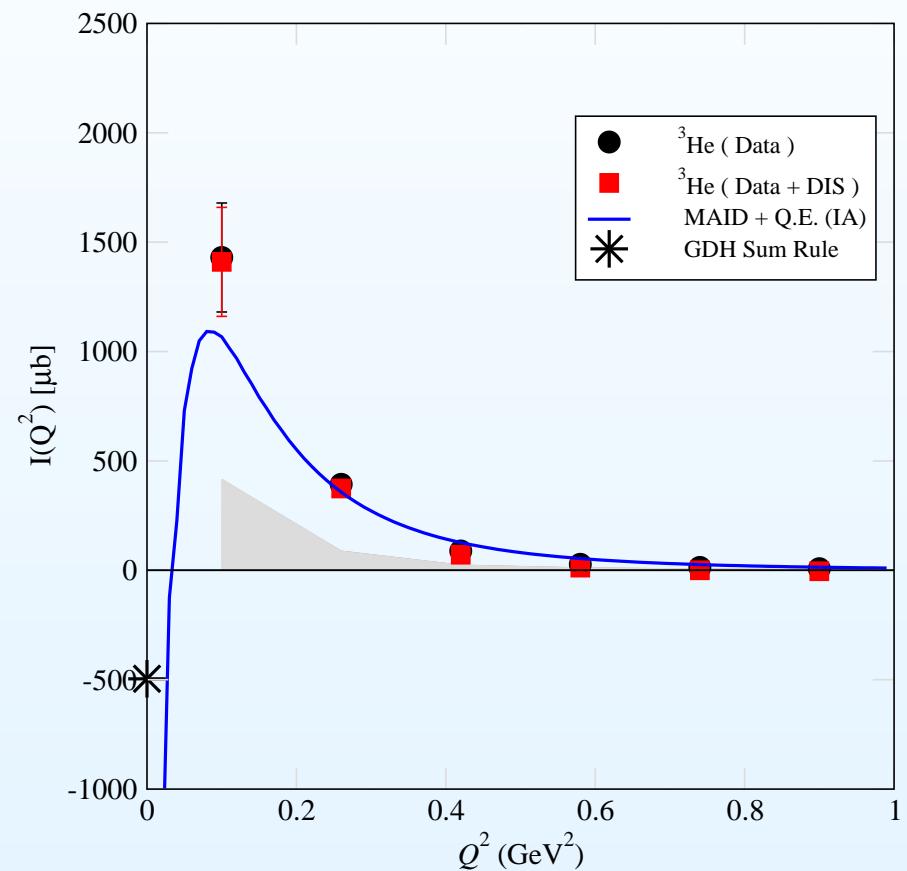
Neutron



M. Amarian *et al.*, PRL 89, 242301 (2002)

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Helium-3

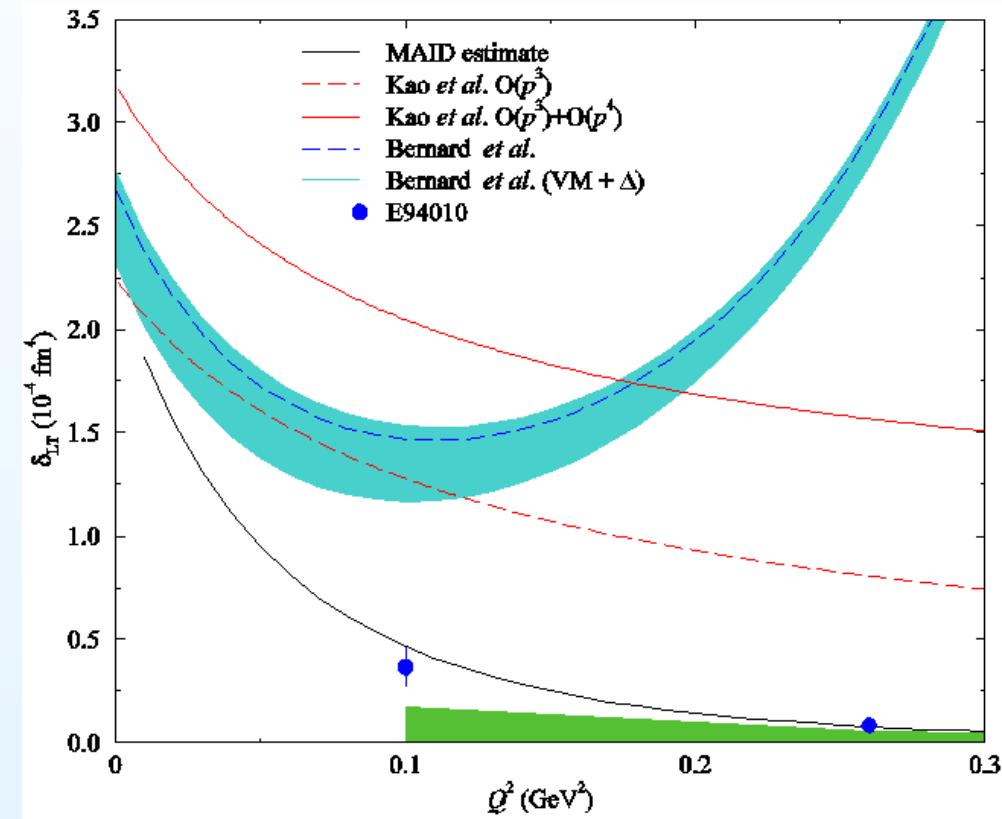
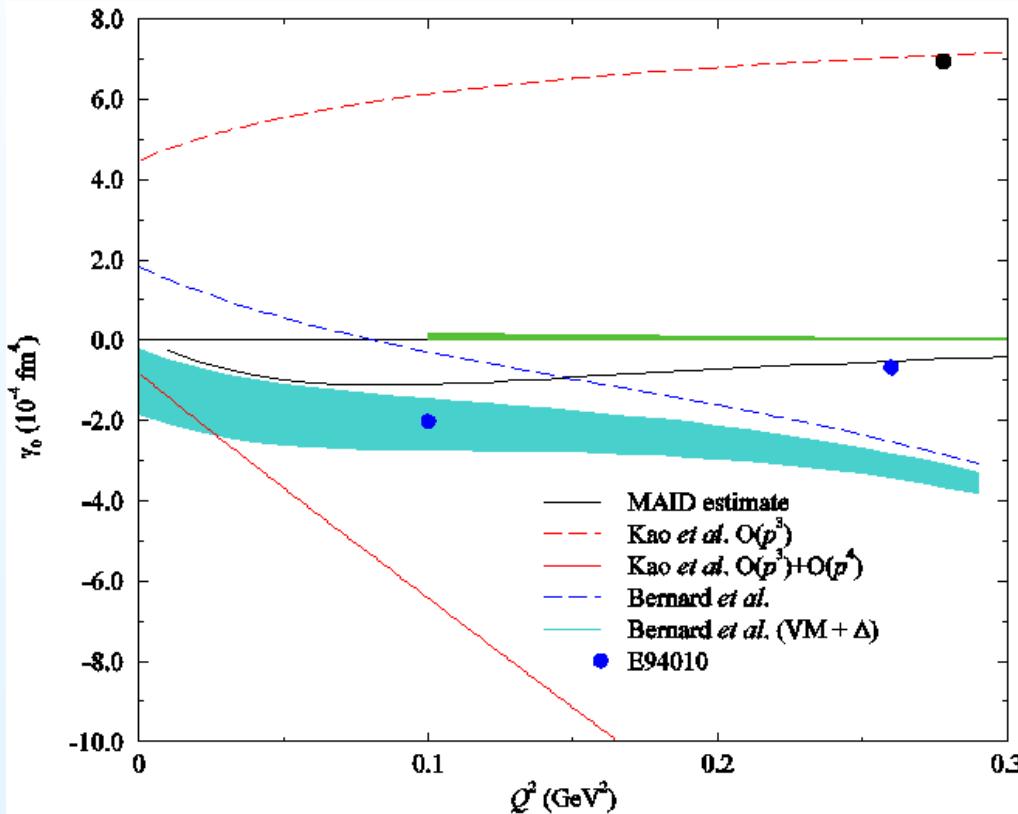


K. Slifer *et al.*, PRL 101, 022303 (2008).

Neutron Spin Polarizabilities

$$\gamma_0 = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left(g_1 - \frac{4M^2}{Q^2} x^2 g_2 \right) dx$$

$$\delta_{LT} = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 (g_1 + g_2) dx$$

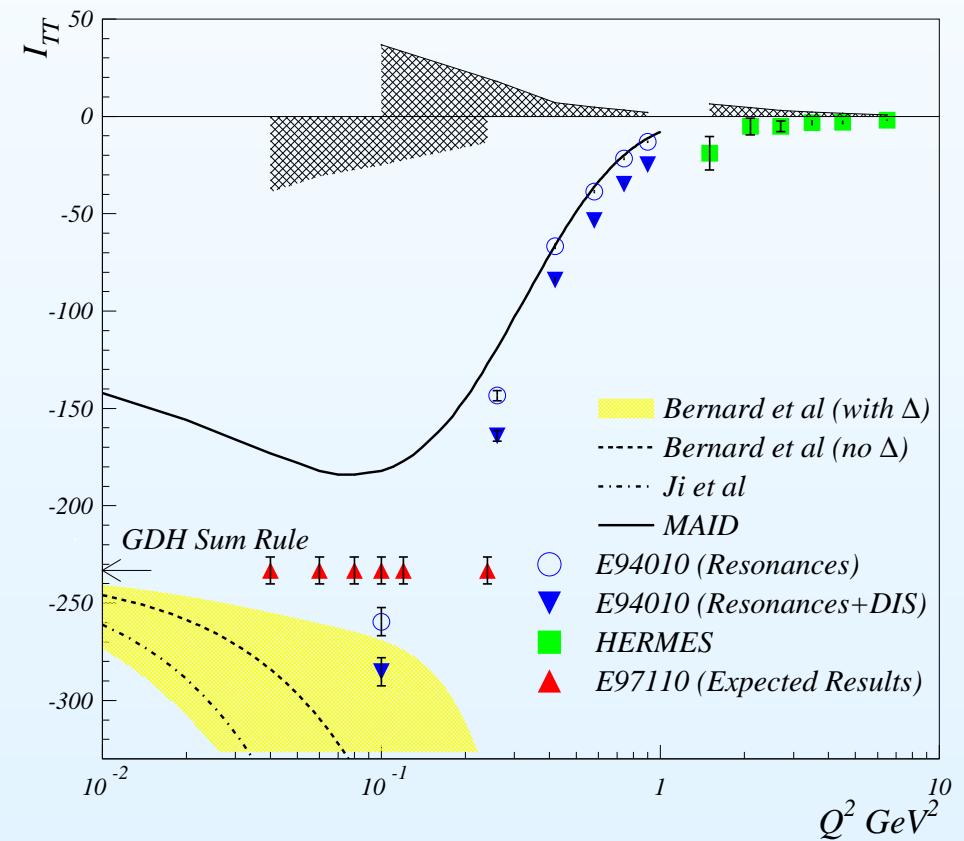


Δ resonance suppressed for δ_{LT}
more robust prediction of χ PT

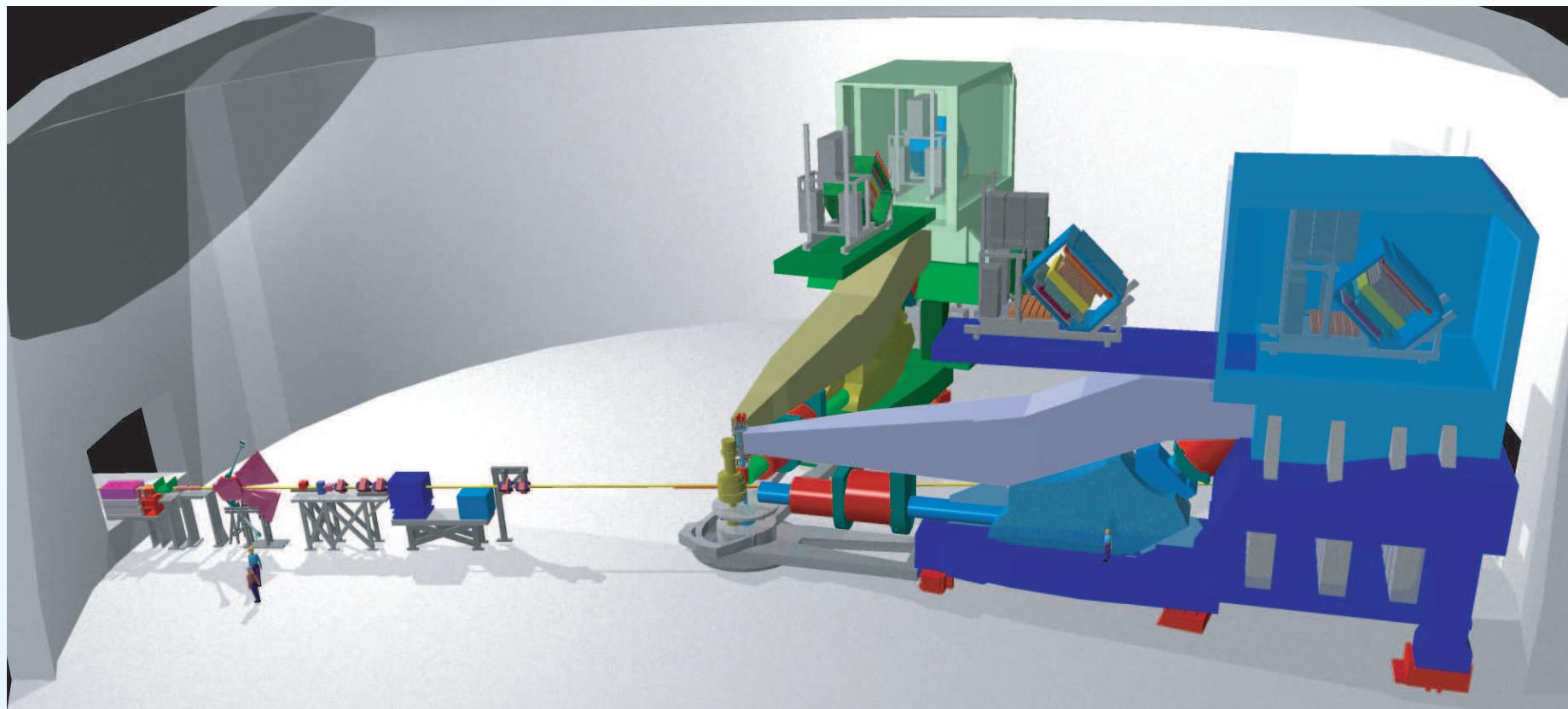
Experiment E97-110

Precise measurement of generalized GDH integral at low Q^2 , 0.02 to 0.25 GeV^2

- Inclusive experiment: $\overset{\rightarrow}{\text{He}}(\vec{e}, e')X$
⇒ Scattering angles of 6° and 9°
- Polarized electron beam:
 $\langle P_{\text{beam}} \rangle = 75\%$
⇒ Pol. ${}^3\text{He}$ target (para & perp):
 $\langle P_{\text{targ}} \rangle = 40\%$
- Measured polarized cross-section differences

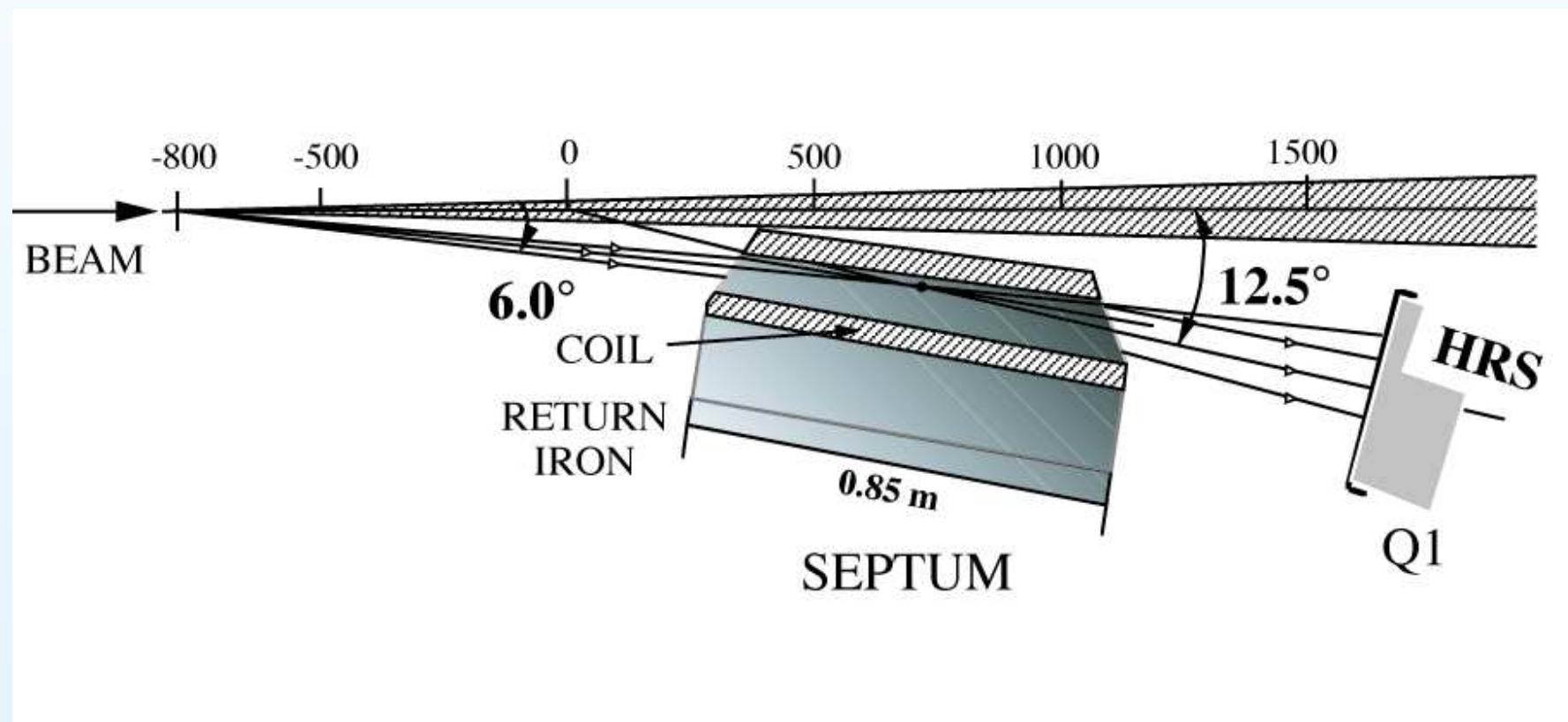


Experimental Setup

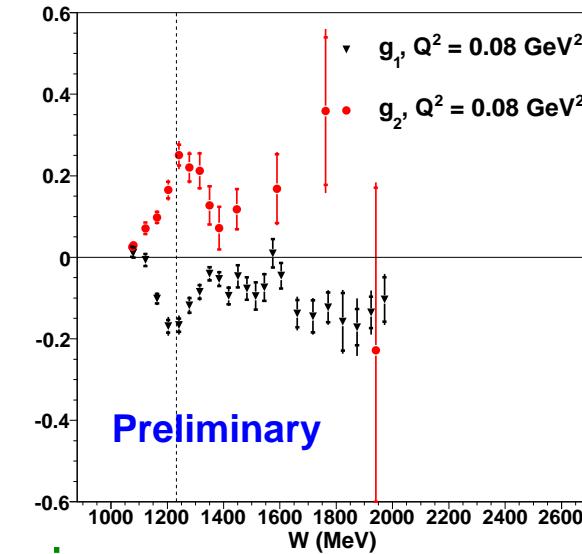
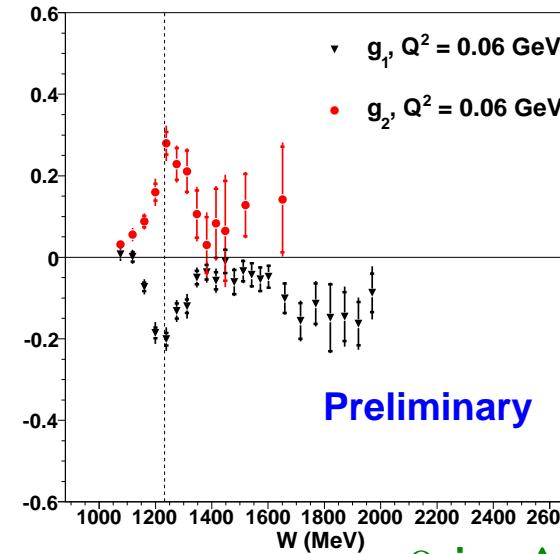
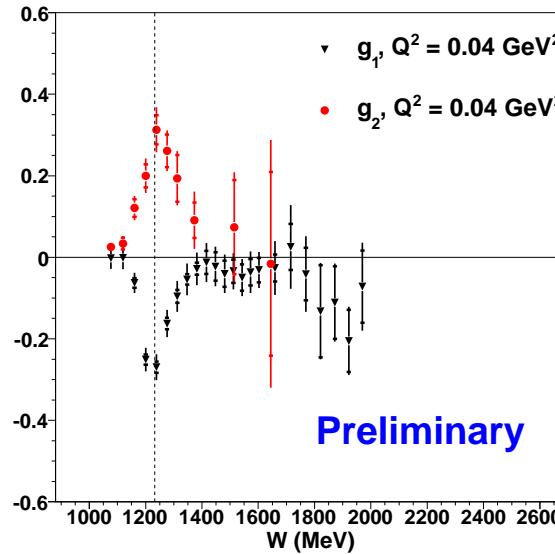


New Bending Magnet

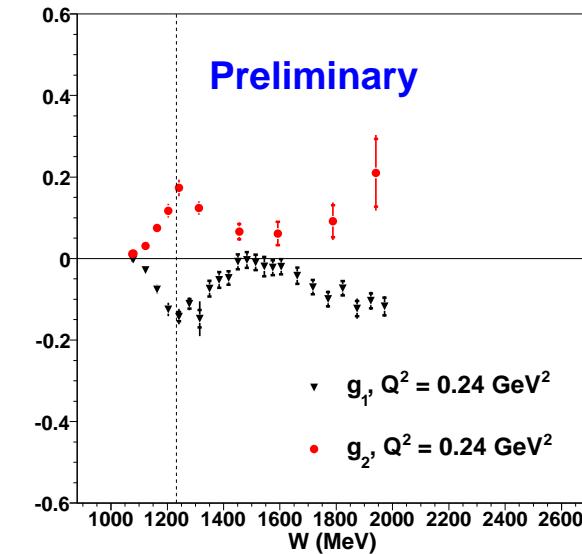
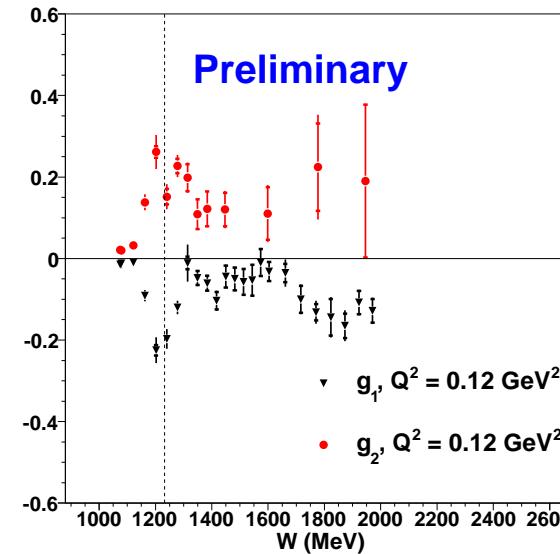
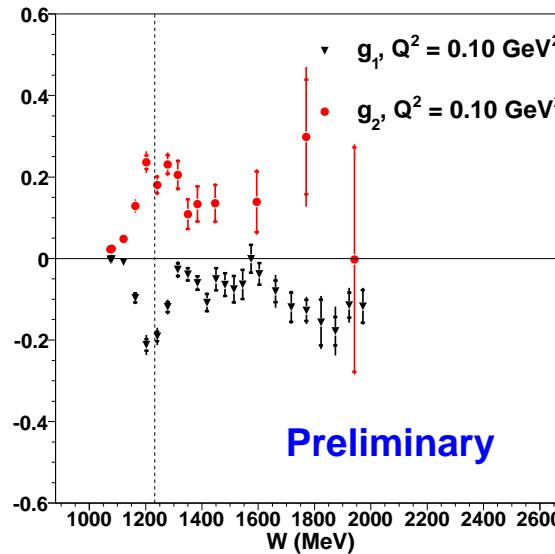
- Low Q^2 requires forward angles.
- Minimum spectrometer angle is 12.5° .



${}^3\text{He}$ - g_1, g_2 versus W at constant Q^2

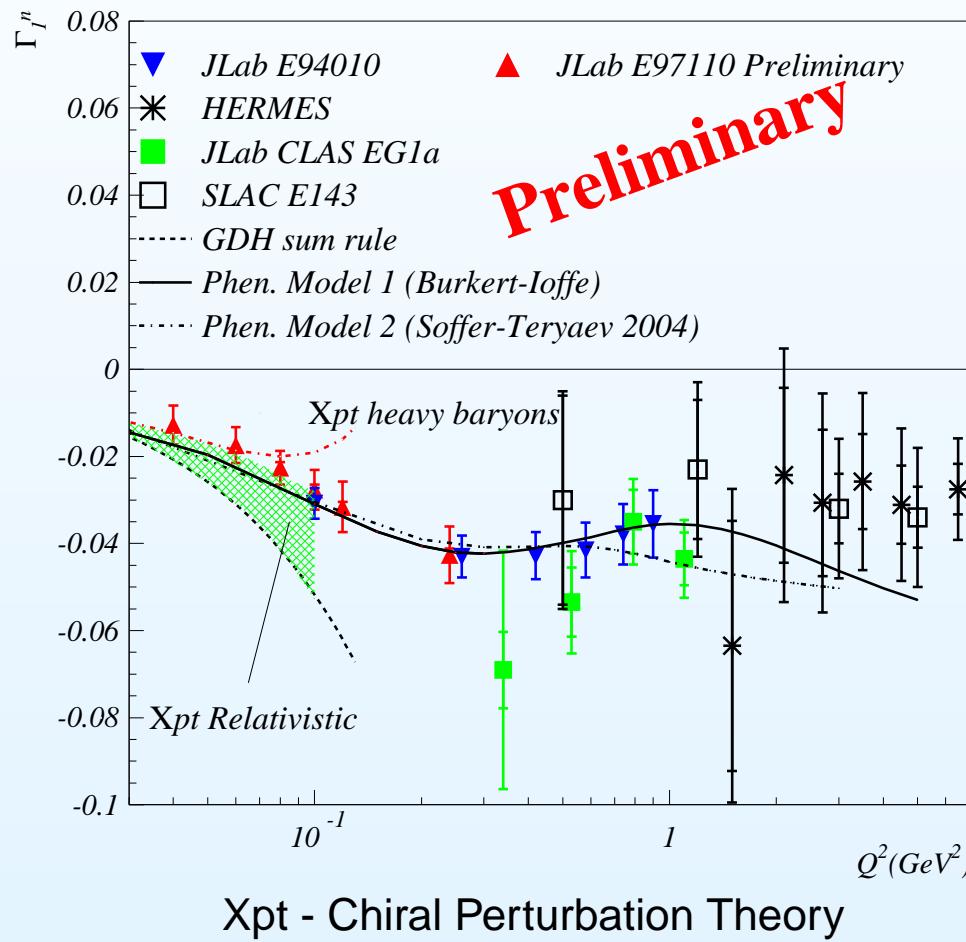


$g_2 \approx -g_1 \Rightarrow \sigma_{LT} \approx 0$ in Δ region



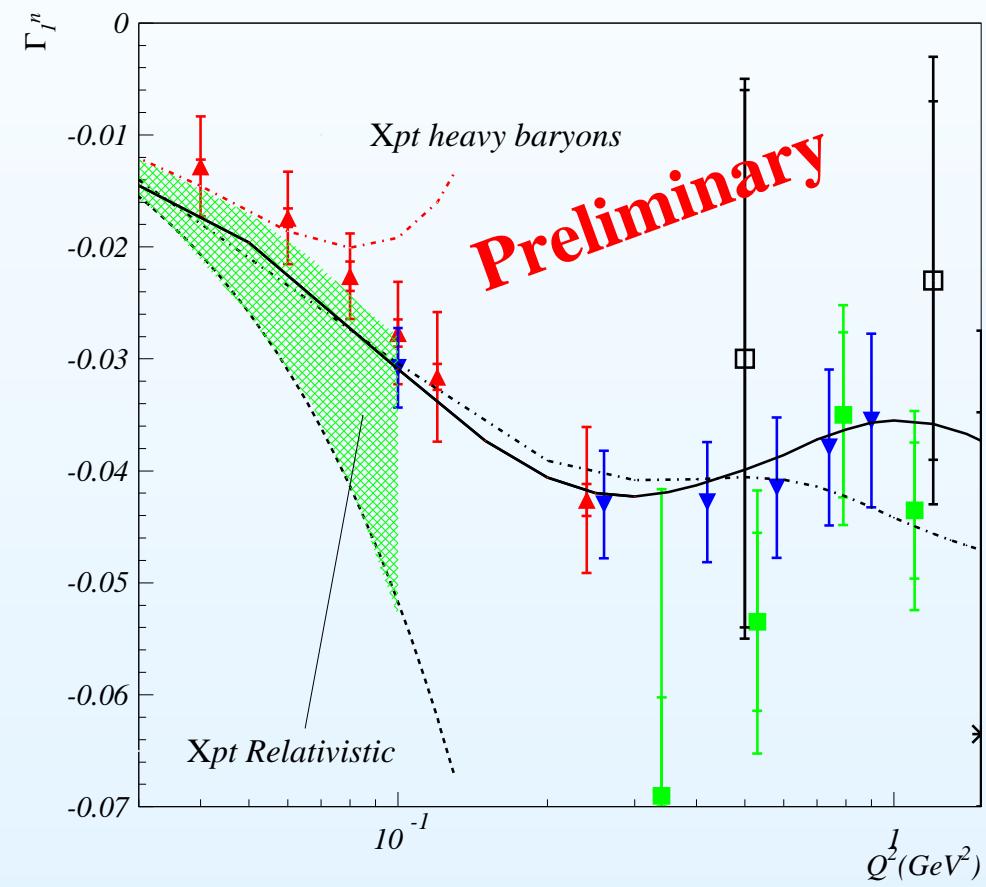
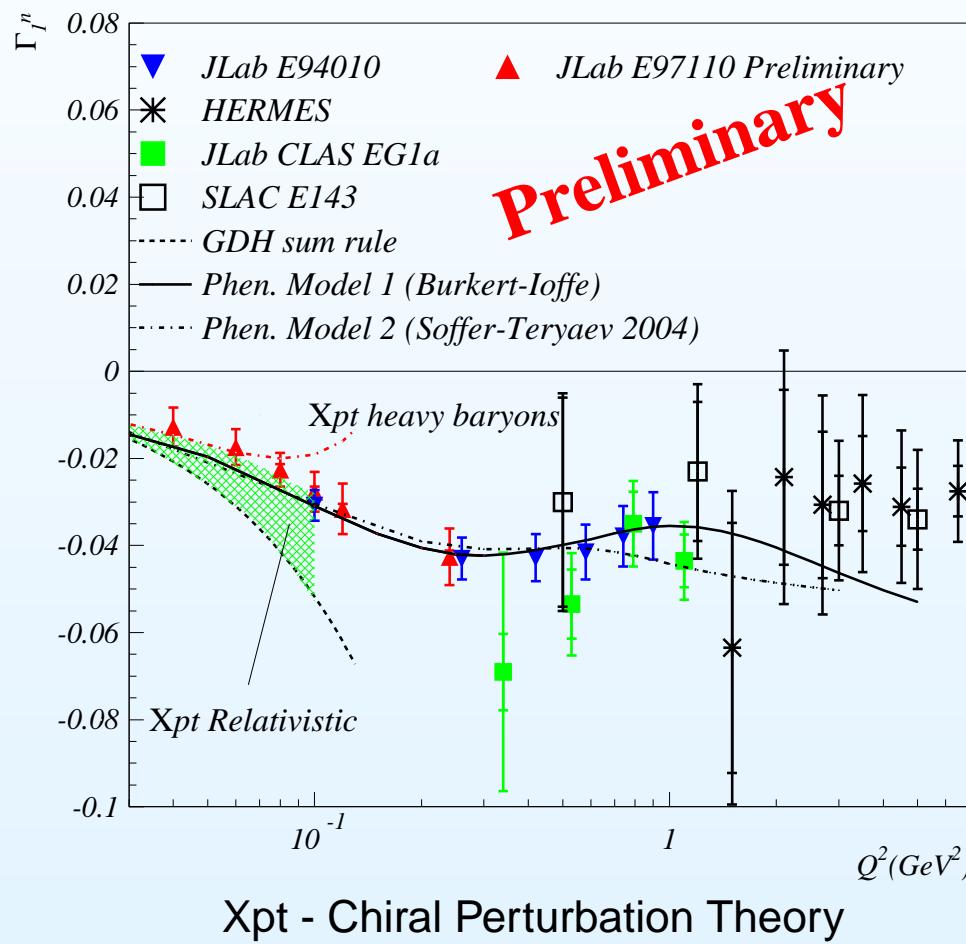
First Moment of g_1

$$\Gamma_1 = \int_0^{x_0} g_1(x, Q^2) dx$$

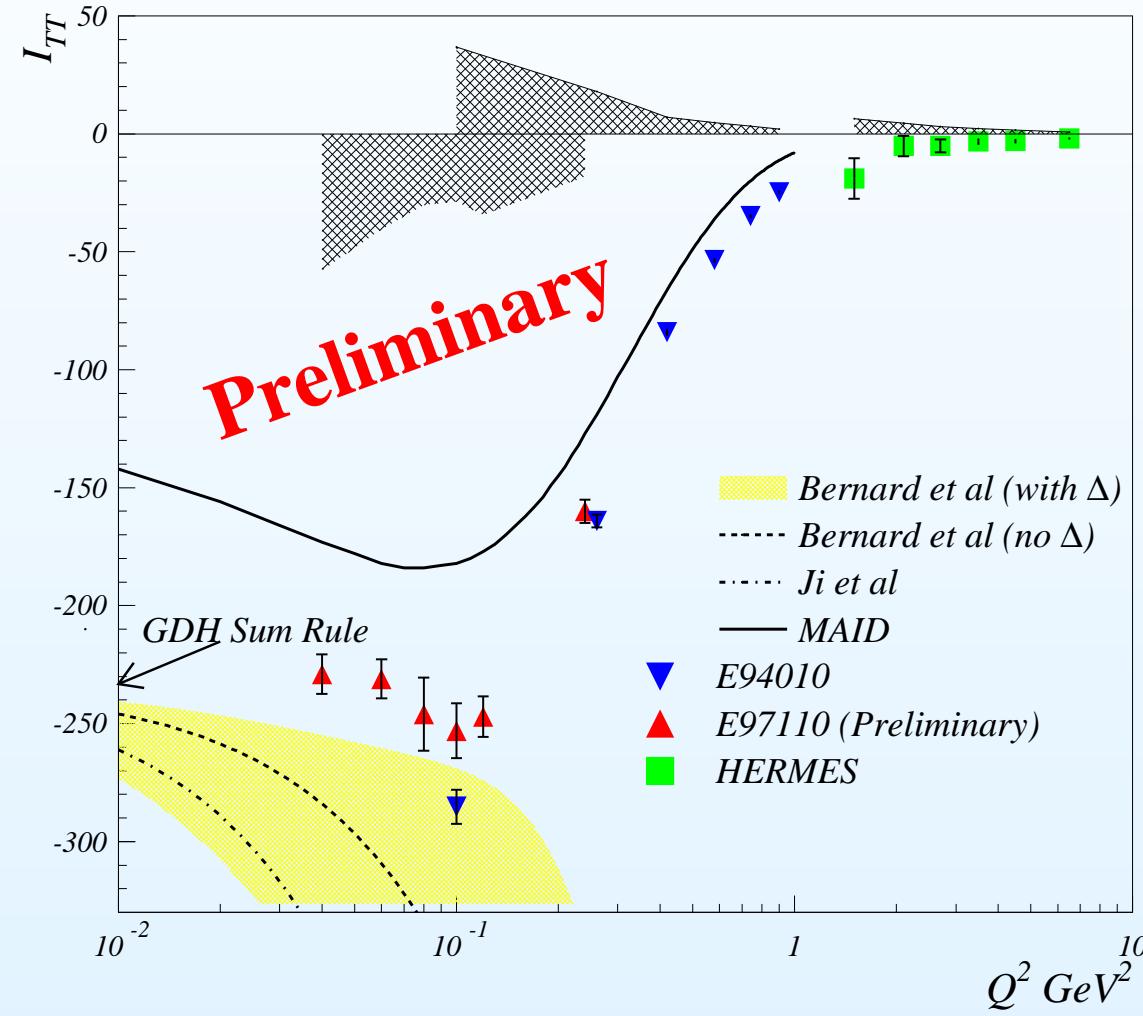


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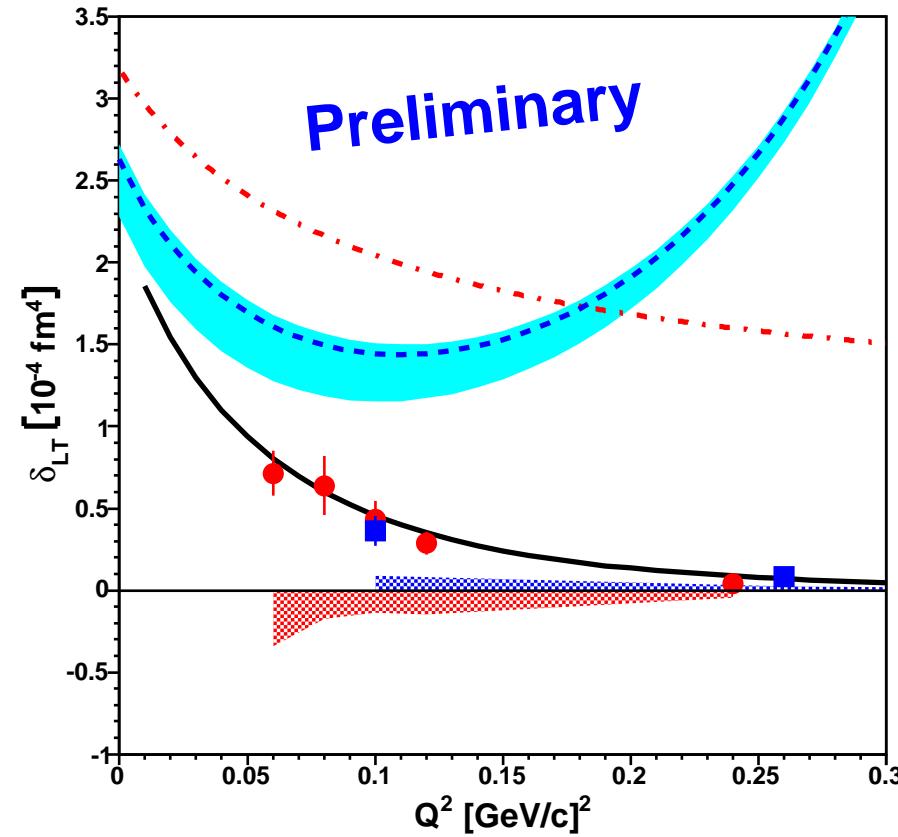
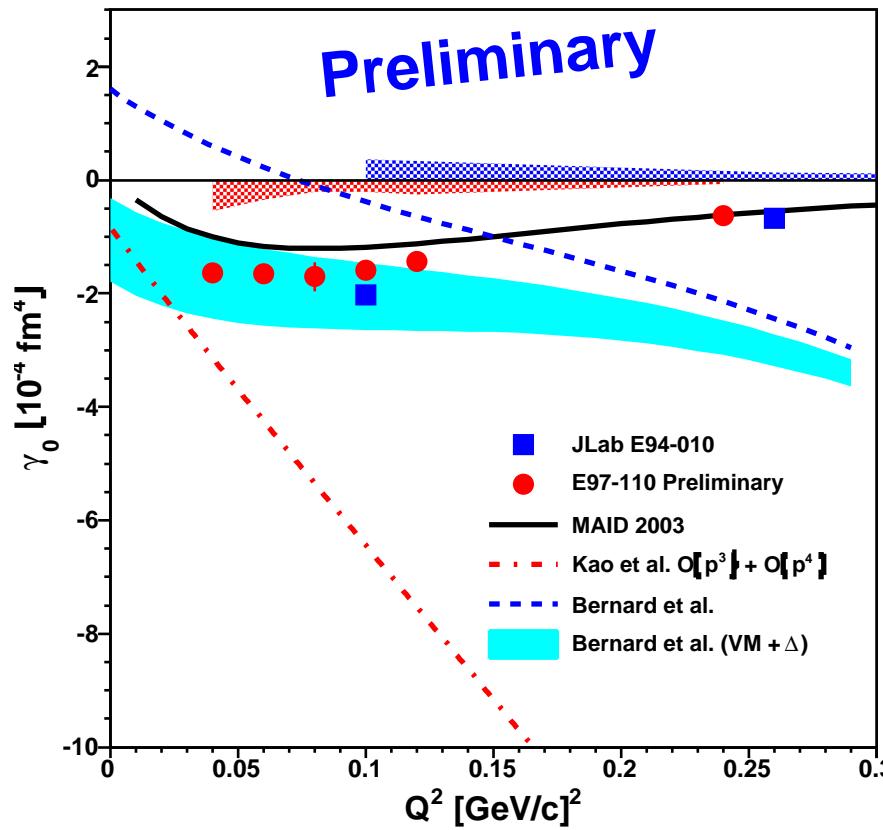
$$\Gamma_1 = \int_0^{x_0} g_1(x, Q^2) dx$$



Neutron I_{GDH}

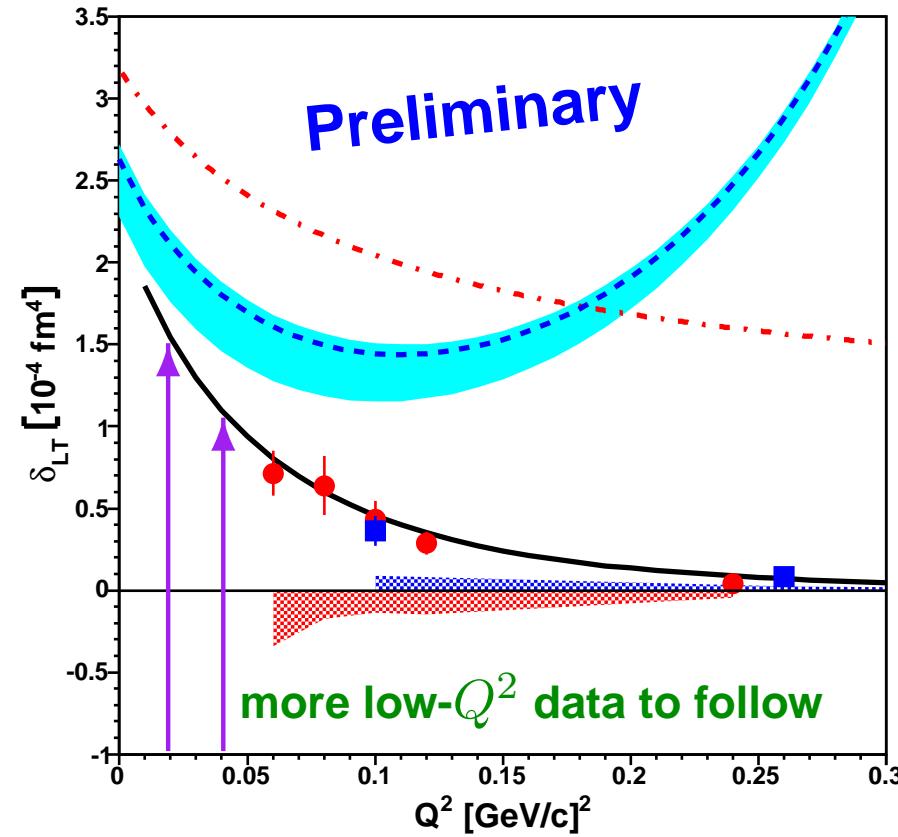
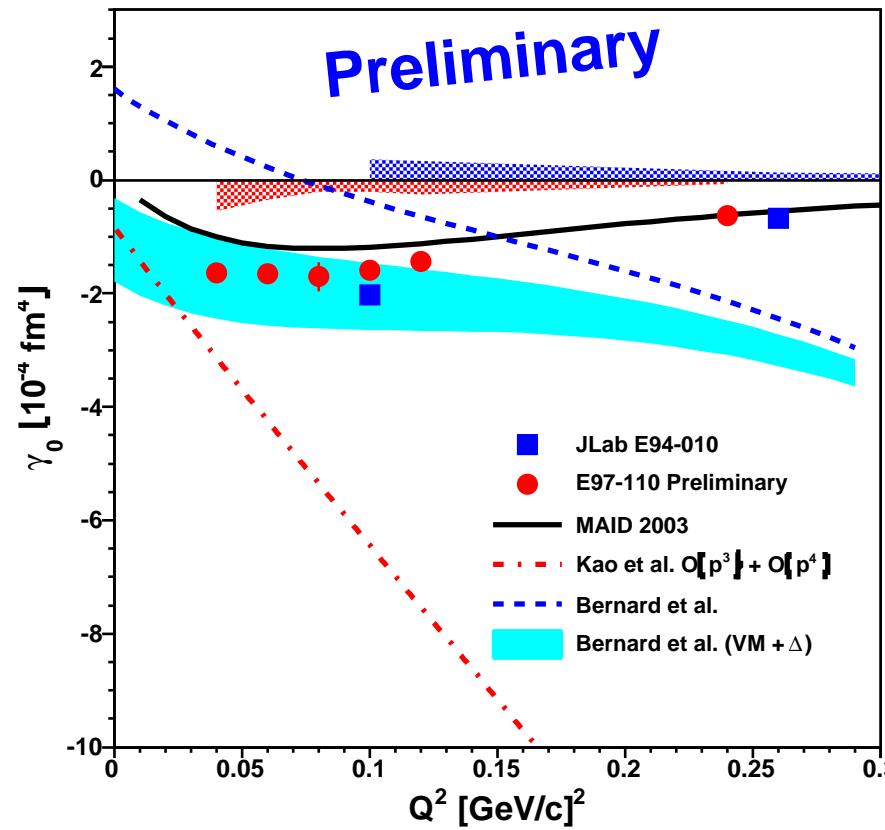


Spin Polarizabilities New Results



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Summary and Conclusion

- The GDH integral is an important tool that can be used to study nucleon spin structure over the full Q^2 range:
 - in particular, the transition from **perturbative QCD** to **nonperturbative QCD**.
- Experiment E97-110 provides precision data for **moments of the spin structure functions** at low Q^2 : 0.02 to 0.25 [GeV/c] 2
- Preliminary results of the **the neutron moments are available** and work is in progress to finalize the systematic effects.
- These data provide a **precise-benchmark test** of Chiral Perturbation Theory calculations at a Q^2 where they are expected to be valid.
- Expect final neutron results soon.

The E97-110 Collaboration

S. Abrahamyan, K. Aniol, D. Armstrong, T. Averett, S. Bailey,
P. Bertin, W. Boeglin, F. Butaru, A. Camsonne, G.D. Cates,
G. Chang, **J.P. Chen**, Seonho Choi, E. Chudakov, L. Coman,
J. Cornejo, B. Craver, F. Cusanno, R. De Leo, C.W. de Jager,
A. Deur, K.E. Ellen, R. Feuerbach, M. Finn, S. Frullani,
K. Fuoti, H. Gao, **F. Garibaldi**, O. Gayou, R. Gilman,
A. Glamazdin, C. Glashausser, J. Gomez, O. Hansen, D. Hayes,
B. Hersman, D. W. Higinbotham, T. Holmstrom, T.B. Humensky,
C. Hyde-Wright, H. Ibrahim, M. Iodice, X. Jiang, L. Kaufman,
A. Kelleher, W. Kim, A. Kolarkar, N. Kolb, W. Korsch,
K. Kramer, G. Kumbartzki, L. Lagamba, G. Laveissiere,
J. LeRose, D. Lhuillier, R. Lindgren, N. Liyanage, B. Ma,
D. Margaziotis, P. Markowitz, K. McCormick, Z.E. Meziani,
R. Michaels, B. Moffit, P. Monaghan, S. Nanda, J. Niedziela,
M. Niskin, K. Paschke, M. Potokar, A. Puckett, V. Punjabi,
Y. Qiang, R. Ransome, B. Reitz, R. Roche, A. Saha, A. Shabetai,
J. Singh, S. Sirca, K. Slifer, R. Snyder, P. Solvignon, R. Stringer,
R. Subedi, **V. Sulkosky**, W.A. Tobias, P. Ulmer, G. Urciuoli,
A. Vacheret, E. Voutier, K. Wang, L. Wan, B. Wojtsekowski,
S. Woo, H. Yao, **J. Yuan**, X. Zheng, L. Zhu

and the Jefferson Lab Hall A Collaboration



Extra Slides



MENU 2010

Jefferson Lab

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Summary of Data Comparison with χ PT

White: no data available
red: poor agreement
yellow: some agreement
green: good agreement

Not sensitive to unmeasured high energy part.

	Γ_1	γ_0	δ_{LT}	d_2
Proton	$a^{exp}=4.31\pm 0.31\pm 1.36$ $a^{ji}=3.89$ Up to $Q^2 \sim 0.08 \text{ GeV}^2$	red	<i>No low Q^2 data</i>	<i>No low Q^2 data</i>
Neutron		Up to $Q^2 \sim 0.1 \text{ GeV}^2$ (Bernard <i>et al.</i> only)	red	red
P-N	$a^{exp}=0.80\pm 0.07\pm 0.23$ $a^{ji}=0.74, a^B=2.4$ Up to $Q^2 \sim 0.3 \text{ GeV}^2$	red	<i>No low Q^2 data</i>	<i>No low Q^2 data</i>
P+N	$a^{exp}=6.97\pm 0.96\pm 1.48$ $a^{ji}=7.11$ Up to $Q^2 \sim 0.1 \text{ GeV}^2$	red	<i>No low Q^2 data</i>	<i>No low Q^2 data</i>

$\xrightarrow{\text{No } \Delta}$

Systematic Uncertainties

Source	Systematic Uncertainty		
	6°	9°	3.775 GeV, 9°
Angle			
Target density		2.0%	
Acceptance	5.0%	5.0%	15.0%
VDC efficiency	3.0%	2.5%	2.5%
Charge		1.0%	
PID efficiency		< 1.0%	
$\delta\sigma_{\text{raw}}$	6.4%	6.2%	15.5%
Nitrogen dilution		0.2–0.5%	
$\delta\sigma_{\text{exp}}$	6.5%	6.3%	15.5%
Beam Polarization		3.5%	
Target Polarization		7.5%	
Radiative Corrections*	20% (40% for $Q^2 \leq 0.08$)		
Total on $\Delta\sigma$	12.1%	12.0%	18.6%

* Radiative correction uncertainty $\approx 6\%$ in delta region

Quark-Parton Model

Infinite-momentum frame ($Q^2 \rightarrow \infty$):

- Partons: **quarks and gluons** (point-like).
- With no quark-quark or quark-gluon interactions.
- x : **fraction of nucleon's momentum** carried by the struck quark
- $q_i(x)$ are quark momentum distribution functions of flavor i .
- \uparrow (\downarrow) quark spin parallel (antiparallel) to nucleon spin.
- $g_2(x)$ related to **quark-gluon correlations**.

$$F_1(x) = \frac{1}{2} \sum_i e_i^2 f_i(x); \quad g_1(x) = \frac{1}{2} \sum_i e_i^2 \Delta q_i(x)$$

$$f_i(x) = q_i^\uparrow(x) + q_i^\downarrow(x); \quad \Delta q_i(x) = q_i^\uparrow(x) - q_i^\downarrow(x)$$

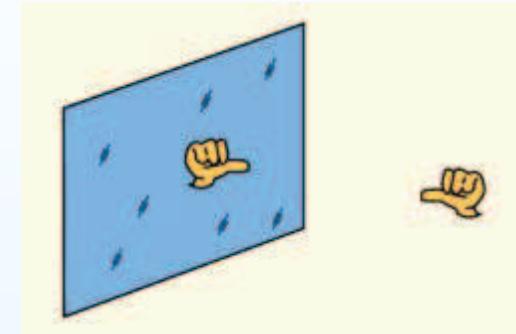
$$F_2(x) = 2x F_1(x); \quad g_2(x) = 0$$

Chiral Symmetry

Chiral symmetry is an important symmetry of the strong interaction.

$m_{u,d} \sim \text{few MeV} \ll M_N \sim 940 \text{ MeV}$:

Chiral symmetry is a fairly accurate symmetry of QCD, i.e. $\frac{M_{u,d}}{M_N} \sim 5 \cdot 10^{-3}$.
 $M_q \neq 0$ explicitly breaks the symmetry.



mirror image of object \neq object
 \Rightarrow object is **chiral**

- Treat light quark masses as perturbations to the chiral symmetric part of QCD
- Makes calculations possible in the non-perturbative region, i.e., Chiral perturbation theory.

GDH Derivation for Real Photons

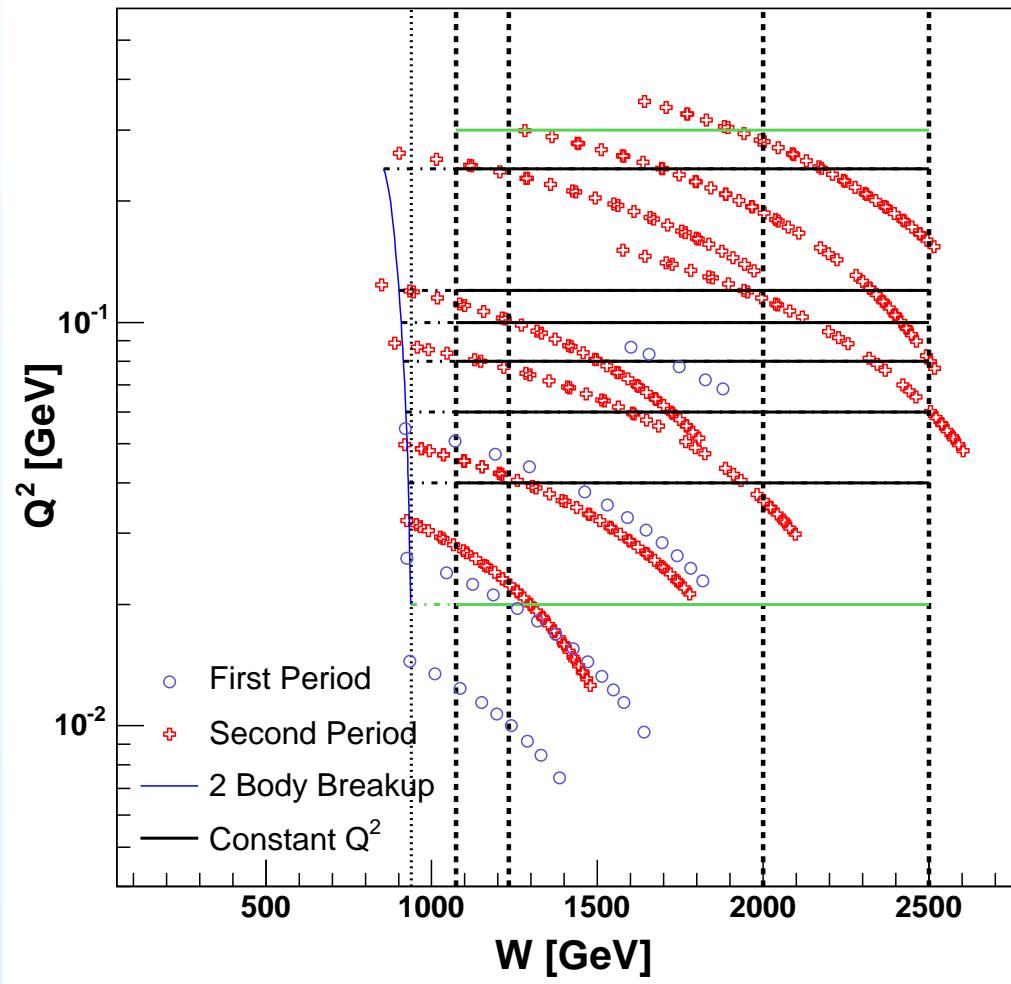
- Begin with the spin dependent part of the forward Compton amplitude, S_1
- Use the following dispersion relation and three assumptions:

$$\text{Re } S_1(\nu) = \frac{2\nu}{\pi} \int_{\nu_{\text{th}}}^{\infty} d\nu' \frac{\text{Im } S_1(\nu')}{\nu'^2 - \nu^2}$$

- Optical Theorem: $\text{Im } S_1(\nu) = \frac{\nu}{8\pi} \sigma_{TT}(\nu)$
- Low Energy Theorem: $\text{Re } S_1(\nu) = -\frac{e^2 \kappa^2}{8\pi M^2} \nu$
- Unsubtracted Dispersion Relation: assumption is convergence of the dispersion integral.

$$I_{\text{GDH}} = \int_{\nu_{\text{th}}}^{\infty} \frac{\sigma_{\frac{1}{2}}(\nu) - \sigma_{\frac{3}{2}}(\nu)}{\nu} d\nu = -2\pi^2 \alpha \left(\frac{\kappa}{M}\right)^2$$

Kinematic Coverage and Interpolation



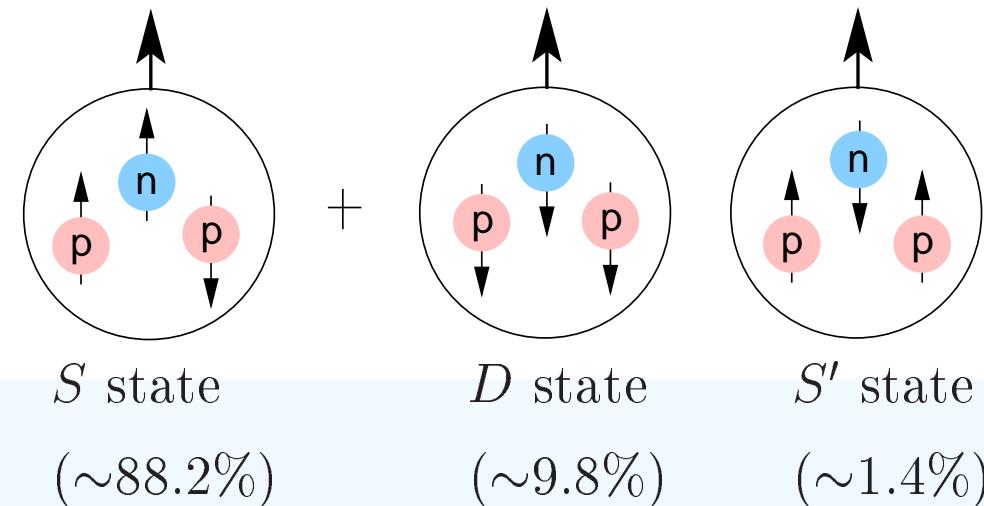
Six constant Q^2 points: 0.04, 0.06, 0.08, 0.1, 0.12 and 0.24 GeV 2 .

Constant Q^2 Interpolation and Integral Extraction

Procedure:

- First interpolate to constant W for each energy.
- Second interpolation with respect to Q^2 .
- Integrals formed from $W = 1073$ GeV to 2000 GeV.
- We could **use our own data above $W = 2000$ GeV**.
- DIS contribution included up to $W = \sqrt{1000}$ using **Thomas and Bianchi parameterization**.
- Neutron extraction performed using calculation from Scopetta and Ciofi degli Atti for $Q^2 \geq 0.1$ GeV 2 .
- $Q^2 < 0.1$ GeV 2 use **effective polarization technique** (difference $\sim 5\text{--}10\%$).

^3He as an Effective Polarized Neutron Target



$$P_n = 86\% \text{ and } P_p = -2.8\%$$

J.L. Friar *et al.*, PRC **42**, (1990) 2310

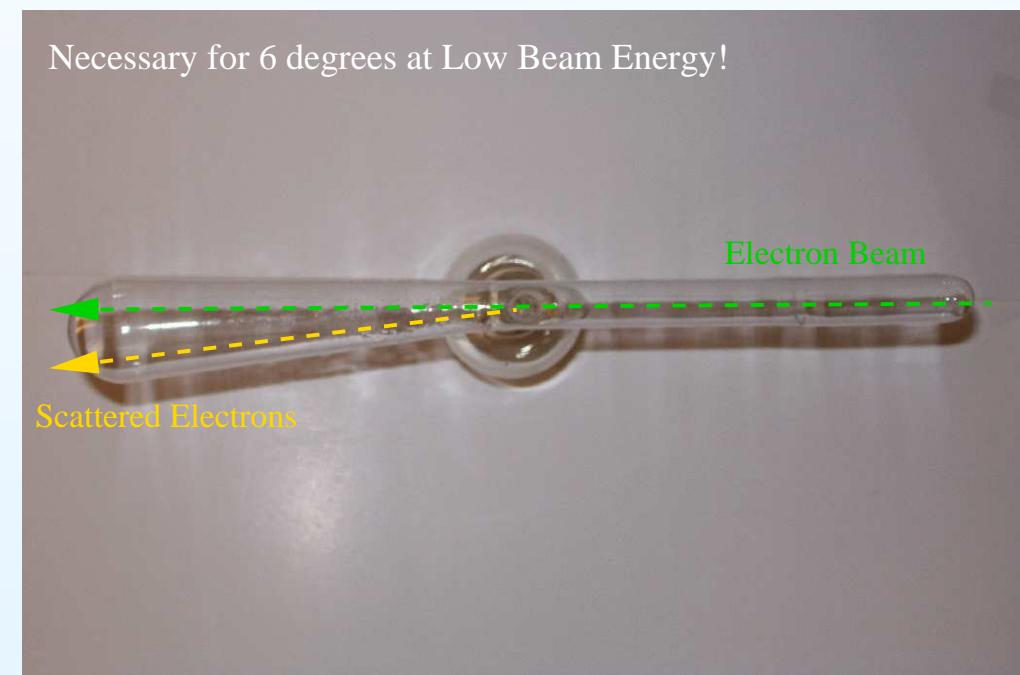
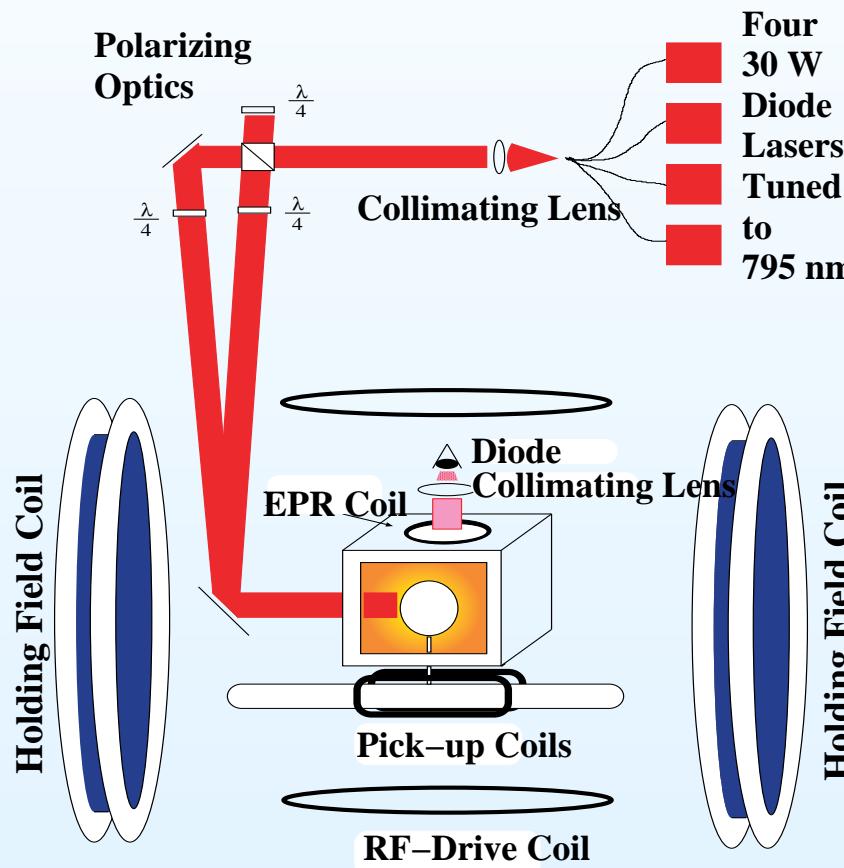
Extraction of Neutron Results

$$\Gamma_1^n(Q^2) = \frac{1}{P_n} [\Gamma_1(^3\text{He}(Q^2)) - 2P_p\Gamma_1^p(Q^2)]$$

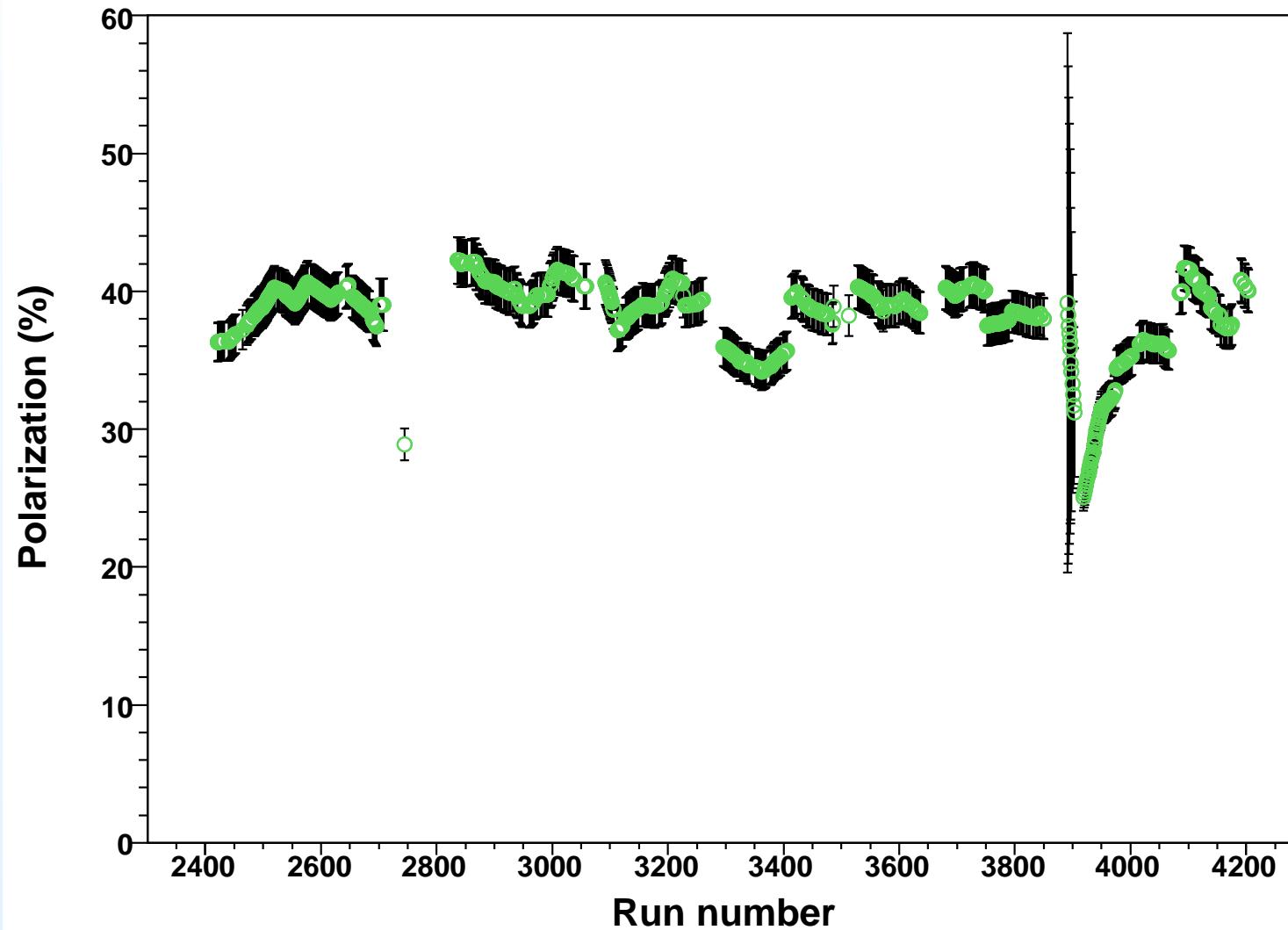
C. Ciofi degli Atti & S. Scopetta, PLB **404**, (1997) 223

Polarized ^3He System

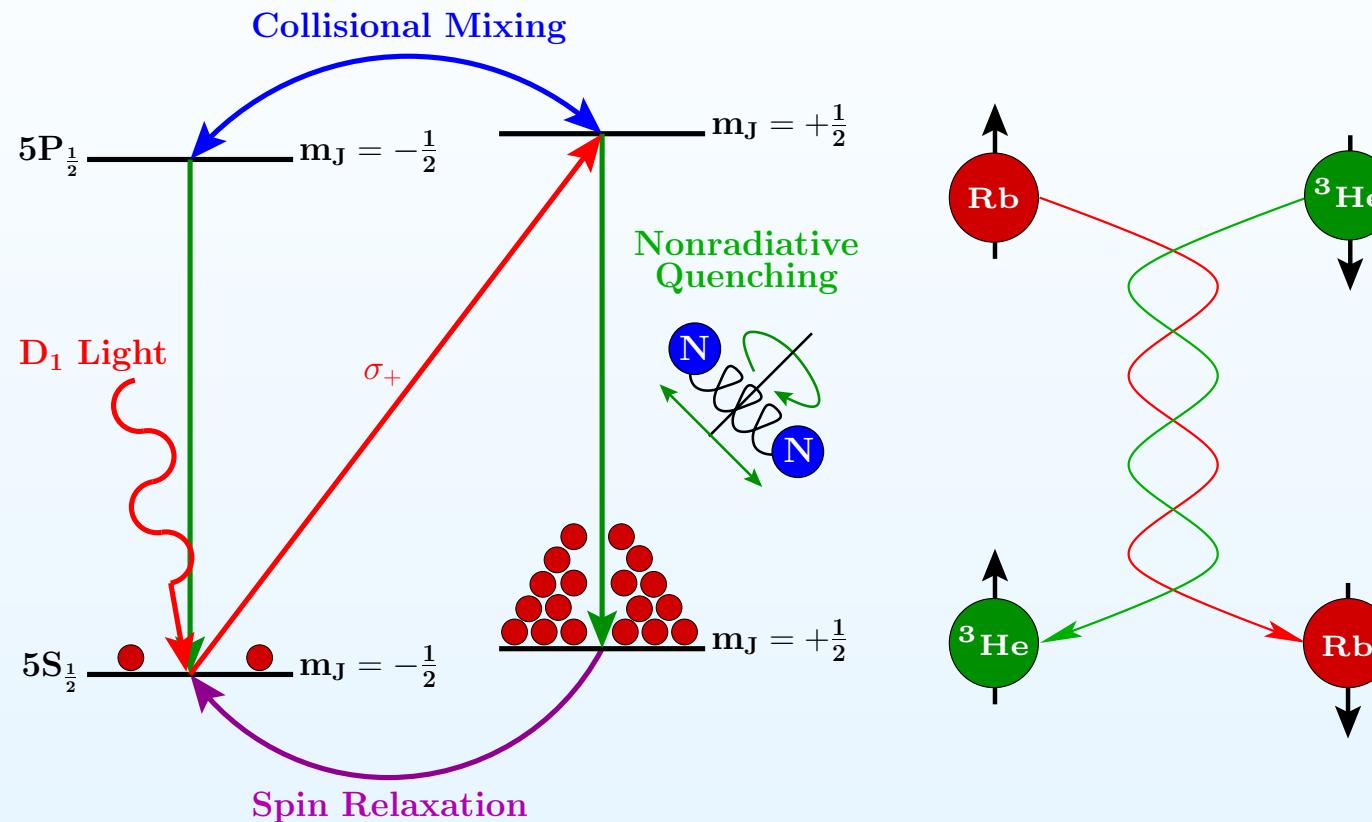
- Both **longitudinal** and **transverse** configurations.
- Two independent polarimетries: **NMR** and **EPR**.



Near Final Target Polarization



Spin Exchange Optical Pumping

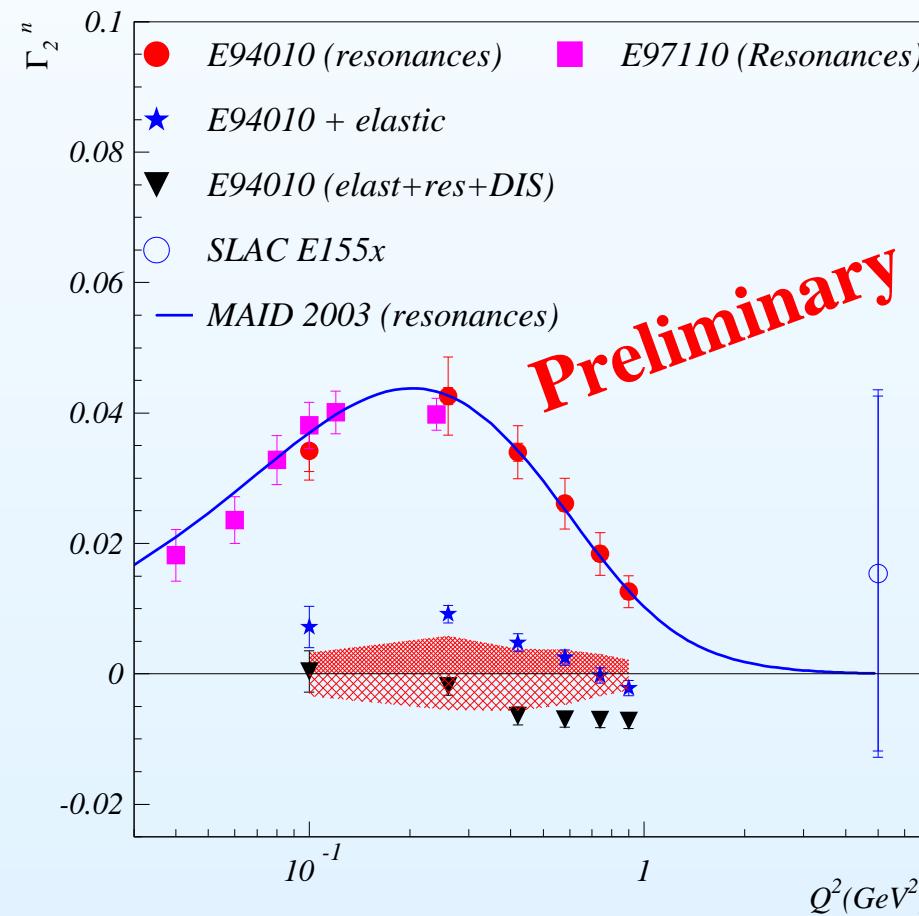


3He nucleus is polarized via **spin-exchange** with optically pumped Rb atoms.

First Moment of g_2

$$\Gamma_2^n(Q^2) = \int_0^1 g_2(x, Q^2) dx = 0$$

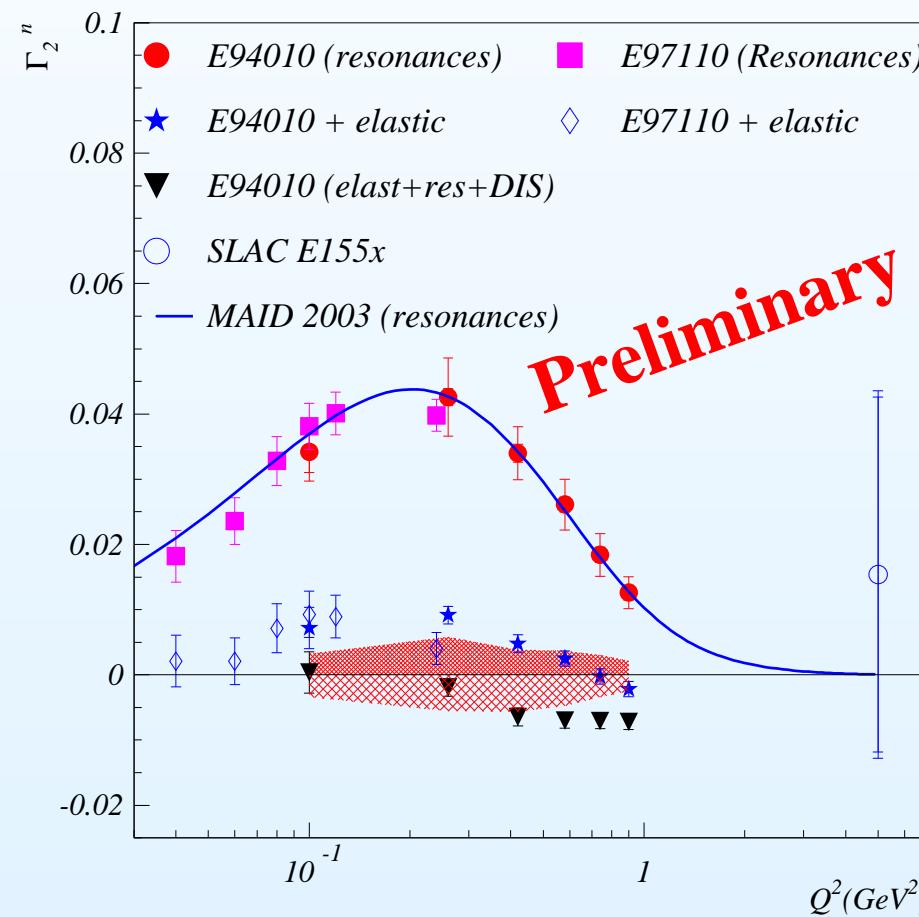
Burkhardt-Cottingham Sum Rule



First Moment of g_2

$$\Gamma_2^n(Q^2) = \int_0^1 g_2(x, Q^2) dx = 0$$

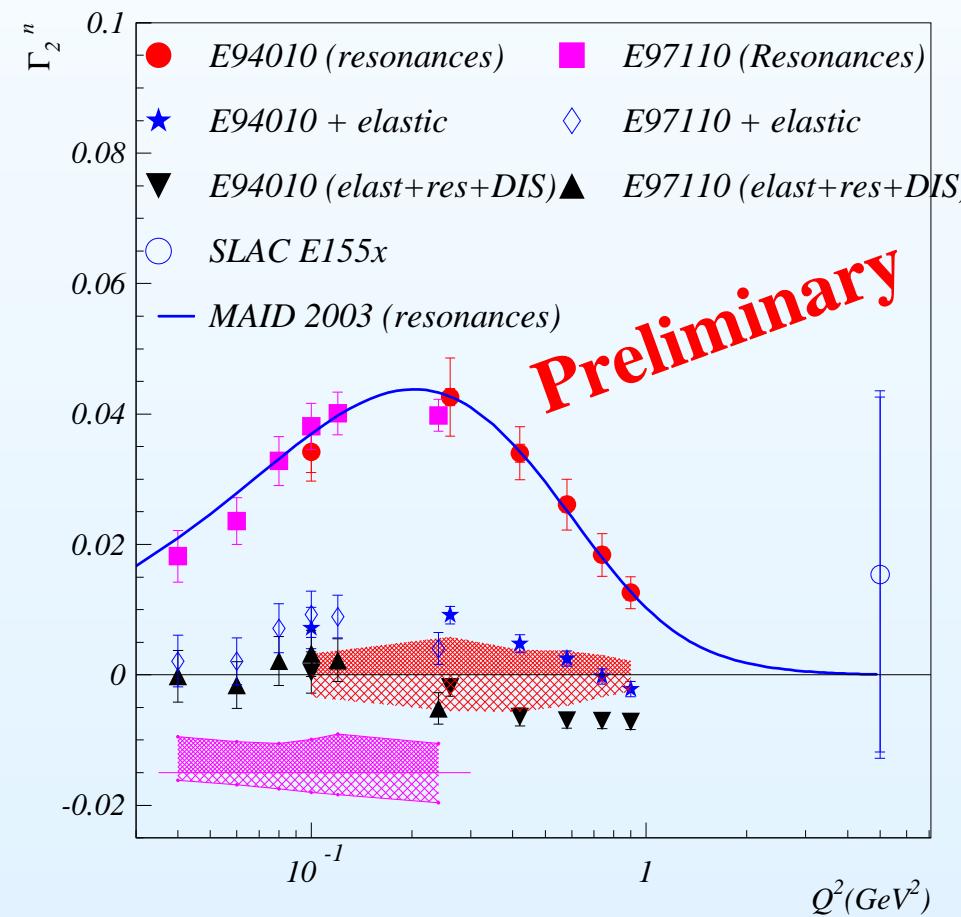
Burkhardt-Cottingham Sum Rule



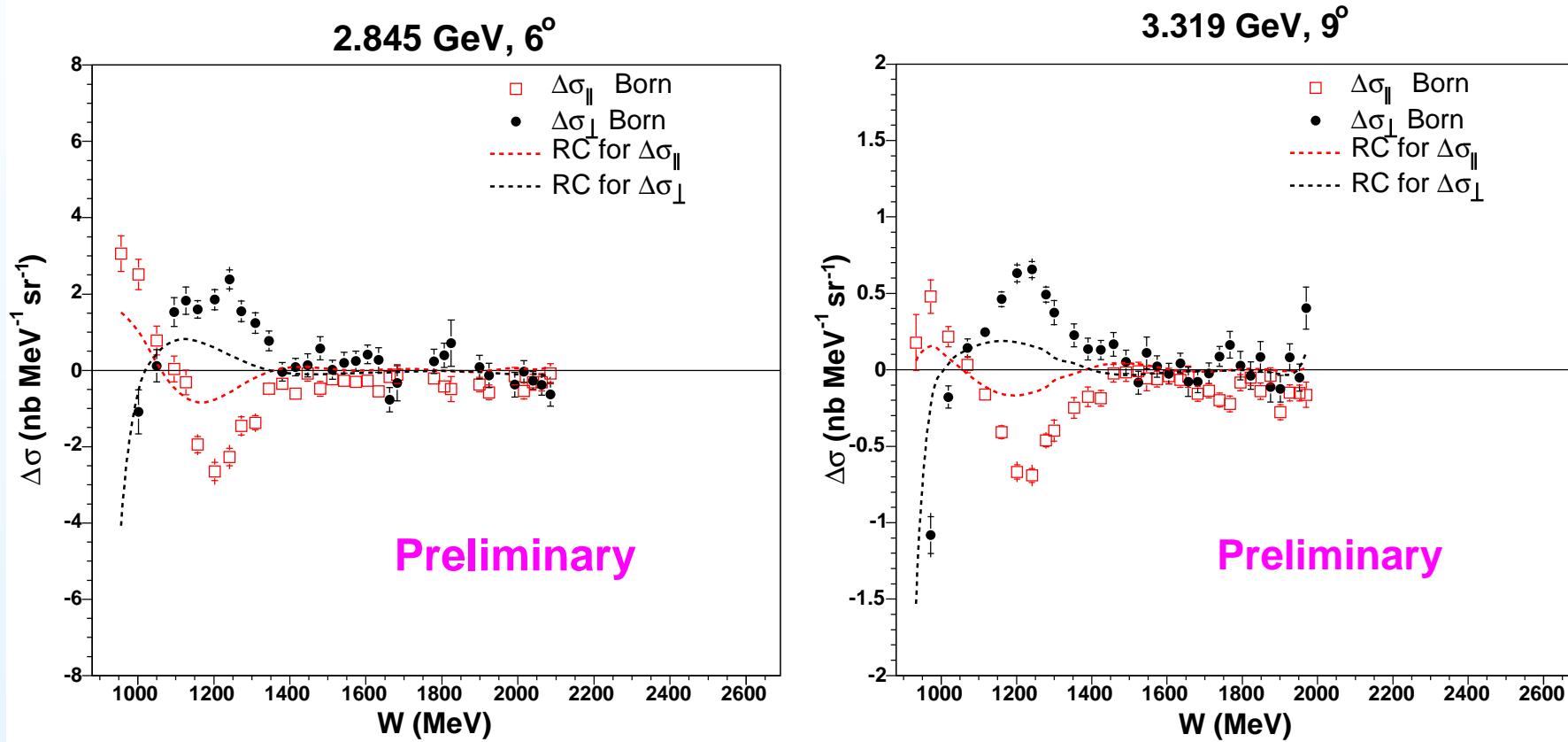
First Moment of g_2

$$\Gamma_2^n(Q^2) = \int_0^1 g_2(x, Q^2) dx = 0$$

Burkhardt-Cottingham Sum Rule



Cross Section Differences



Radiative corrections: formalism of L. Mo and Y. Tsai (unpolarized) and POLRAD (polarized), work done by J. Singh.